ṬUSI, NAṢIR-AL-DIN II. AS MATHEMATICIAN AND ASTRONOMER

ṬUSI, NASIR-AL-DIN

ii. As a Mathematician and Astronomer

Naṣir-al-Din Ṭusi participated in the whole spectrum of mathematical sciences that were known in antiquity: from elementary works on arithmetic to the more advanced works on geometry, and to what we would now call mathematical geography and spherical trigonometry (which were then considered to be an introduction to astronomy), to astronomy proper in all its branches, as well as to its junior sister, the astrological science, and the related fields of optics and trigonometry. In all of these fields and subfields, Ṭusi's work could be characterized as innovative at least, even when he was just dealing with re-editions of mathematical and astronomical texts of antiquity, and extremely creative at its best, when it involved his own analysis of fresh topics that were either unknown or unresolved by the ancients.

*Ṭusi's Taḥrir*s. Ṭusi seems to have embarked on his own education in the classical mathematical sciences by producing working versions in Arabic of existing masterpieces, texts that he systematically called *Taḥrir*s, a difficult term to translate exactly, but which could be approximately rendered by "redaction." In most cases, what he would do was to take all the available Arabic translations of a given text, and then re-write it in his own version by relying on all the earlier translations and choosing from them the words that he thought best brought out the intended scientific meaning of the text. Wherever he detected a failure in translation, he would not hesitate to substitute his own words for theirs. Next, he would study the contents of the text itself and try to improve upon them by avoiding, for example, repetitions, which were in some instances quite frequent, and would also update them by using contemporaneous concepts and techniques rather than holding on to ancient concepts and methods.

The text that best illustrates Ṭusi's method in this regard is his *Taḥrir al-majesṭi* (Redaction of the *Almagest*; Saliba, 1987). There he replaced the chord calculations that were used by Ptolemy in the earlier chapters of the *Almagest* by their more current trigonometric equivalents that Ṭusi knew were widely accepted and used in his time. In the later books of the *Almagest*, where Ptolemy repeated, for every planet, the same iterative method to determine the equants of the various planets (i.e., for each planet, the actual point at some distance from the earth about which its epicycle revolved), Ṭusi went ahead and gave one example of the method, and then applied it to the various planets by simply inserting and varying the particular parameters of each.

In a sense, the concept of the taḥrir itself, as developed and applied by Ṭusi, was a somewhat idiosyncratic though extremely useful type of a new scientific composition. He felt free to use the earlier ideas, add to them, embellish them, edit them in the modern sense of editing, rearrange and rephrase them as he saw fit, and finally supply his own wording for specific concepts that would express more coherently the intended meaning or would improve upon it, so that the text would become more relevant to his own era. This technique also allowed Ṭusi to make comments of his own that were either pertinent to the original contents of the text, or to the Arabic language of the translation. In the same *Taḥrir al-majesṭi*, he would say, for example, that such and such a concept in the original text was difficult to follow, or hard to believe, and would then invite the reader to look into it further, or he would promise that he himself would return to it in his own later writings. As for the language, he would often say that other translations gave this or that choice of words instead of the one that he had opted for, and would also at times leave both readings side by side for the reader to compare and choose from. If, however, the text was definitely problematic, as in the case of Ptolemy's description of the latitudinal motion of the planets in the thirteenth book of the *Almagest*, Ṭusi would not hesitate to interject and record his own displeasure with the text by saying, in this particular instance: *hāḏā kalām(on) ḵārej(on) ʿan al-ṣenʿa* "This kind of speech does not belong to the craft" [of astronomy]. Immediately following that, he would go on to give his own views as to how the text could be improved upon and in that case would even suggest a new theorem that he would later develop on his own, and use to solve other unrelated problems.

We shall return to this peculiar technique and development in Ṭusi's thoughts below. For now, it should be sufficient to have had those particular examples of the manner in which Ṭusi interfered with the contents of the classical scientific texts in order to illustrate his general approach, and to indicate the extent to which modern students of Ṭusi's texts should strive to isolate Ṭusi's own ideas at each point from those already expounded by classical authors. The technique followed by Ṭusi makes this task quite difficult, and requires one to place the original classical Greek text next to the final version of Ṭusi at each juncture, in order to appreciate the radical reformulations that those texts went through at his hands. This is a necessary task in order to understand the very technical and advanced nature of the developments that Ṭusi instigated. Starting with the *Elements* of Euclid, and ending with the *Almagest* of Ptolemy, passing through the so-called *motawassaṭāt* (The Middle Books), such as the works of Autolycos, Aristarchos, Theodosius, Apollonius, Archimedes, Hypsicles, and Menelaus, among others, which were always considered to fall in between those two magisterial texts throughout antiquity, he seems to have produced a taḥrir of each and every classical mathematical or astronomical text he could lay his hands on. In a sense, he seems to have recast the whole library of classical mathematical sciences. It is indeed unfortunate that none of those "new" productions has ever been properly edited and studied by modern scholars. But the great numbers of surviving manuscripts, scattered in major and minor libraries across the world, do indeed attest to the popularity of those texts among the students of the mathematical sciences during the long, productive life of Islamic civilization.

In some instances, Ṭusi's texts fully supplanted the originals and as a result it became harder and harder to locate the original counterparts. One could easily find many more manuscript copies of Ṭusi's *Taḥrir al-majesṭi*, for example, than copies of either of the original Arabic translations of the *Almagest*. When, towards the end of the sixteenth century, the printing of the Arabic version of Euclid's *Elements* was undertaken by the Oriental Medici

Press in Florence, the decision was taken not to go back to the ninth-century Arabic translations of Euclid, although at least two of them were extant, but to opt for a hybrid text of a much later date that embodied Ṭusi's taḥrir instead.

Ṭusi's independent mathematical works. When it comes to works that he had to produce from scratch, his corpus also reveals a whole spectrum of mathematical works that range from the mundane, popular types to the sheer ingenious, creative works that left their mark on human history in the succeeding centuries. In his *Jawāmeʿ al-ḥesāb be'l-taḵt wa'l-torāb* (arithmetical compendium using board and dust; Saidan, 1967), for example, which he apparently completed in 1264, some ten years before he died, he attempted to describe arithmetical operations at a popular level, this time by combining the traditional Greek number theory (i.e., classical arithmetic) with the Indian tradition that came to be known in Islamic times as *al-Ḥesāb al-hendi* (Indian arithmetic). In doing so he was not departing radically from the works of his predecessors since the tenth century. Like them, he too was simply making arithmetical operations accessible to a wider public. On that level alone this work could be conceived as an example of Ṭusi's concern for popular education in the arena of arithmetical sciences.

But he did more than that. He actually took a step beyond what was done before, namely by introducing a "new" method for the extraction of roots that was already known in much older Chinese sources that do not seem to have been known to Ṭusi, and which was later rediscovered by Blaise Pascal (d. 1662). He also devoted a special section of the *Jawāmeʿ al-ḥesāb* to what he called the arithmetic of the astrologers, in contradistinction to the arithmetic of the calculators, to which he also devoted a special, separate section. In the section on astrological calculations he explained how arithmetical operations could be brought to bear on such sexagesimal entities as signs, degrees, minutes, and seconds that were normally used by the astrologers. This section could also be perceived as an introduction to astronomical operations in general, as the astronomers too used the same sexagesimal quantities as their astrological colleagues. It was a forerunner of the kind of arithmetical works that would later treat the subject much more elaborately, as was done by Kāši (d. 1429) and others.

What stands out about *Jawāmeʿ al-ḥesāb*, however, is Ṭusi's choice for the title, which indicates a clear use of the dust-board at this late date, when paper and paper-making had already proliferated all over the Muslim world and even reached the European mainland. Ṭusi's predecessors, and chief among them

Oqlidesi (before 952 CE), had already urged some three centuries earlier that arithmetic, particularly its Indian variety, be practiced on paper, rather than on the dust-board, for two major reasons. First, arithmetical operations written down on paper could be easily checked for error, in contradistinction to those drawn with a stylus (*mil*) on a dust-board, which were successively erased as the operations proceeded. Second, Oqlidesi had also argued that one should avoid using the dust-board for the simple social reason that such boards were associated with street astrologers, who used them to cast horoscopes for passers-by.

Why would Ṭusi then go back to writing a book on Indian Arithmetic, with the ingenious innovations that he had brought to it, and why would he specifically describe all the operations that had to be carried out on a dust-board? The only reason that springs to mind is that Ṭusi was probably systematically adhering to his desire to popularize arithmetic, and had therefore decided to persevere with such tools of the trade for pure economic reasons: Dust was always cheaper than paper at all times and in all places, and was easily available to everyone. Ṭusi himself, however, has given no such explicit justification, either in the introduction to the book, or in the rest of the detailed chapters on arithmetical operations.

It is indeed unfortunate that the other mathematical works of Ṭusi remain unedited and are therefore rarely studied by modern scholars. As a result, very little can be said about them, except to underline that, based on what we already know from his studied works, a serious investigation of their contents should prove fruitful. In fact, whenever something new becomes known about them, it always appears to be very suggestive and seminal.

His participation, for example, in the debates which had raged for centuries in Islamic civilization about the nature of the Euclidian Fifth Postulate (the “Parallel Postulate”) is of the highest order, and together with the contribution of others kept this discussion on the foundations of geometry alive up to the 18th and 19th centuries, when it finally led to the discovery of non-Euclidian geometry. Ṭusi’s own correspondence with ʿAlam-al-Din Qayṣar (d. 1251; Rażawi, 1975, pp. 371 f.), his classmate and colleague from the days of their studentship under the tutelage of Kamāl-al-Din b. Yunos (d. 1242), constitutes an essential chapter in this debate, and the insights he exhibited in the discussion remain truly remarkable even to a modern reader (Rosenfeld et al., 1996, chap. 14).

As mentioned above, his work on trigonometry must have started with his redactions of the *Almagest*, and the book of Menelaus. In his redaction of the *Almagest* we saw that he replaced the two chord theorems of the *Almagest*, those of Menelaus and Ptolemy, with their trigonometric equivalents. In so doing, he must have realized that he was deploying the results of the new field of trigonometry that had developed in Islamic times along lines divergent from those followed in classical antiquity. As a result, Ṭusi decided to devote a special treatise to the trigonometric innovations that he had encountered and deployed in his various taḥrirs. Thus, for the first time in Islamic civilization, he endowed the field of trigonometry with the status of an independent field. The Arabic designation of the Menelaus theorem as *al-Šakl al-qaṭṭāʿ* (the sector configuration), on account of the fact that it dealt with relations among sections of circular arcs on a spherical surface, gave rise to the title of Ṭusi's treatise, as can be easily detected from its general designation in the various manuscripts simply as *Ketābal-šakl al-qaṭṭāʿ* (Book of the sector configuration), or *Kašf al-qenāʿʿan asrār al-šakl al-qaṭṭāʿ* (Unveiling the secrets of the sector configuration). The importance of Ṭusi's work on trigonometry is that it compiles for the modern reader a complete picture of trigonometric relations that are essentially the same as the ones found in modern textbooks on the subject. In a sense, the idea of an independent trigonometric textbook can also be traced to this particular work by Ṭusi.

Ṭusi's independent astronomical works. Ṭusi's astronomical production covered the whole gamut of astronomical activities known in his time: from theoretical works such as his already mentioned taḥrir of the *Almagest*, which, as already pointed out, could be arguably considered an independent work, to *zij*es (astronomical tables—most importantly the *Ilḵāni zij*; see below), to *taqwim*s (records of planetary positions for specific times), to astrological handbooks, and several works that dealt with *hayʾa* (cosmological astronomy), which was then a new, but already well established field (Ragep, pp. 24-75).

It is not clear when Ṭusi first became interested in astronomical disciplines. But it is safe to say that his studentship days under Kamāl-al-Din of Mosul, already mentioned before, may have instigated this interest. We know of Kamāl-al-Din's special concern in matters relating to the mathematical sciences; among his students were such famous figures as ʿAlam-al-Din Qayṣar, also mentioned before, and Aṯir-al-Din Abhari (d. 1262). According to well-attested reports, Kamāl-al-Din even gave lectures on the text of the *Almagest* itself (Suter, pp. 140 f.). Ṭusi was in his twenties when he seems to have made a shift from his study of religious sciences and came to Mosul in order to study mathematical sciences with Kamāl-al-Din. This may be the time when he learned astronomy, in all likelihood through one of the then classical texts on hayʾa, such as the *Tabṣera* of al-Ḵaraqi (d. 1139). Ṭusi's own production of a Persian text only a few years later under the title *Resāla-ye Moʿiniya*, which he dedicated to his Ismāʿili patron Moʿin-al-Din the son of Nāṣer-al-Din Moḥtašemi (d. 1257), is clearly modeled after such a text and, like the *Tabṣera*, dealt with astronomical problems that were more of a descriptive cosmological nature (Dānešpažuh, 1956). Similar to the *Tabṣera*, the contents of the *Moʿiniya* were rather elementary, the type one would start with as a student, and did not deal with the difficult cosmological issues of Greek astronomy that were already known for more than three centuries. Its importance, however, is that it was one of the very few Persian texts ever written on the subject. The only other Persian text that may have been authored before in the hayʾa style is the anonymous text known as *Jehān-šenāḵt,* which may date to a century before the *Moʿiniya*, if not more. Otherwise, the very great majority of such hayʾa texts were written in Arabic, at least up to the middle of the 20th century, even when they were written by Persian-speaking authors.

This elementary nature of the *Moʿiniya* made it inadequate for the more serious students of astronomy, who, by Ṭusi's time, had already heard about the attacks leveled against Greek astronomy by people like Ebn al-Hayṯam (Alhazen, d. 1039), Abu ʿObayd Juzjāni (d. ca. 1075) and the anonymous Andalusian astronomer who lived around the year 1060 and who had authored a text called *Recapitulation Against Ptolemy* (Saliba, 1999).

When he came to write his taḥrir of the *Almagest* in 1247, Ṭusi felt that he should at least make some reference to the thorny points in the Greek astronomical tradition in order to draw the students' attention to them and to register his own awareness of such problems. He would not have been regarded as a serious astronomer if he had not done so. Thus, in the same taḥrir*,* he went much further than issuing simple warnings, taking the already mentioned courageous step when he denounced the Ptolemaic theory of the latitudinal motion of the planets by using such a phrase as: *hāḏā kalām(on) ḵārej(on) ʿan al-ṣenʿa* "This kind of speech does not belong to the craft" [of astronomy]. All this demonstrated that he too was participating in this debate against Greek astronomy and proved him to be more in keeping with the Islamic astronomical tradition of the time, which had almost exhausted the enumeration of the problems in that astronomy. That was probably also why

Ṭusi must have felt obliged, in his own *Redaction of the Almagest*, to introduce for the first time, perhaps, a rudimentary form of a mathematical theorem that is now referred to in the literature as the Ṭusi Couple (FIGURE 1). The essence of the Ṭusi Couple, as stated in this text, is to produce linear motion as a result of two circular motions. That was achieved by allowing two identical circles, one riding on the circumference of the other, and moving in such a way that, when the carrier circle moved at a specific speed in one direction, the other riding circle moved at twice that speed in the opposite direction. This allowed the farthest point on the riding circle, which lies along the line that joins the centers of the two circles, to oscillate in a linear fashion, up and down along the line that joins the centers of the circles, while the two circles moved uniformly around their own centers in the directions just described.

A few years later, perhaps sometime during the 1250s, and certainly sometime between 1247, when he finished the *Redaction of the Almagest*, and 1261, when he finished his next hayʾa text, the *Taḏkera*, Ṭusi's readers of the Persian text of the *Moʿiniya*, and in particular the patron for whom the *Moʿiniya* was written in the first place must have urged him to update the *Moʿiniya*, so that it too would answer some, if not all, the Greek astronomical problems. It was then that he must have composed the addendum that is variably known as *Ḏayl-e Moʿiniya*, *Šarḥ-e Moʿiniya*, or *Ḥall-e moškellāt-e Moʿiniya*. In it he clearly took a much bolder step and developed the theorem that he had only hinted at in the *Redaction of the Almagest* in a rudimentary form and now restated in the more appropriate language of spheres. Instead of two circles he now talked of two spheres, one twice the size of the other, and the smaller one internally tangent to the larger. He then allowed the larger sphere to move at any speed in one direction and the smaller sphere to move at twice the speed in the opposite direction. Ṭusi went on to prove mathematically that the resulting motion of the two spheres just described would force the point of inner tangency to oscillate along the diameter of the lager circle, thus producing the required linear motion as a result of two uniform, circular motions. Furthermore, he also realized by then that this motion could be applied, not only to resolve the latitudinal motion of the planets, but also to solve the problem of the motion of the moon, or what could be called the lunar equant problem, and thus resolve one of the thorniest issues in the Ptolemaic configuration for lunar motion. Those and other problems constituted the bulk of the *Ḏayl*, which was also written in Persian.

By 1260 or so, one of Ṭusi's colleagues, apparently a scholar of some repute but who is otherwise relatively unknown by the name of ʿEzz-al-Din Zanjāni (d. 1261), requested from Ṭusi an Arabic text that would give a summary of his latest thinking on the subject of cosmological astronomy, the discipline that was by then known simply as hayʾa. In response, Ṭusi produced his famous *al-Taḏkera fi ʿelm al- hayʾa* (Memento on the science of astronomy), in which he restated what he had already written in Persian in the *Moʿiniya*, and also alluded to the *Redaction of the Almagest*, and added a special chapter (Book II, chapter 11 of the *Taḏkera*), in which he restated what he had already said in the *Ḏayl* with additions, now calling the chapter *fi'l-ešāra elāḥall mā yanḥall men al-eškālāt al-wāreda ʿalāḥarakāt al-kawākeb al-maḏkura al-lati sabaqat al-ešāra elayhā* (Indicating the solution of that which could be solved of the previously indicated problems that afflict the motions of the aforementioned planets). True to form, he did indeed produce the solution for the motions of the moon and the superior planets. But when it came to the planet Mercury, Ṭusi had this to say: *Ammā fi ʿOṭāred fa-lam yatayassar li tawahhom ḏāleka ka-mā yanbaḡi… wa-in yassara Allāh taʿāla ḏāleka alḥaqtoho be-hāḏa al-maḍiʾ* “As for Mercury, I still have not figured it out properly, and should the Almighty God ever make that possible, I will add it to this place” [in the text]. This statement clearly indicates that up to that point Ṭusi considered his writing as works-in-progress. From then on, we do not know if Ṭusi ever revisited the problem, and if he did, there is no surviving evidence.

It was in this additional chapter of the *Taḏkera* that Ṭusi finally formally stated and proved the Ṭusi Couple that he had first expressed in a rudimentary form in the *Redaction of the Almagest*, and further developed in the *Ḏayl-e Moʿiniya*; and it was in this chapter that he clearly stated that “he was the first to devise” a solution for the lunar motion, for which he had to commence with a lemma, the lemma which is now called the Ṭusi Couple. He was therefore quite aware that his mathematical theorem was unprecedented, and that it could be applied to a variety of problems and was not restricted to the latitudinal theory of the planets for which it was first conceived in the *Redaction of the Almagest*. It was this very mathematical theorem that became his enduring legacy to later astronomers in the Islamic world and must have certainly reached the Latin West, as it has now been shown to have been used in the works of Copernicus (FIGURE 2; Hartner, 1973).

Ṭusi’s other works on theoretical astronomy that could be classified as hayʾa texts, such as his Arabic introduction called *zobdat al-edrāk fi hayʾat al-aflāk* (Solaymān, 1994) or his Persian *zobdat al-hayʾa,* which had its own Arabic translation, are both too elementary to warrant any serious mention alongside the much more mature work, the *Taḏkera*. No wonder, then, that of all the works of Ṭusi on the subject of hayʾa, it was the *Taḏkera* which attracted the most attention from later astronomers, who either wrote their own commentaries on it, as was done by Ṭusi's own student Qoṭb-al-Din Širāzi (d. 1311) and the latter's student Neẓām-al-Din Nišāburi (d. 1328), and by the later astronomer al-Šarif Jorjāni (d.1413), or wrote commentaries on the commentaries, as was done, for example, by Šams-al-Din Ḵafri (d. 1550). The many surviving copies of the *Taḏkera* and the commentaries and commentaries on the commentaries of the same, testify to the popularity of this remarkable work, and probably make it the most widely studied book of astronomy ever written in Arabic. The only competitors in this regard would be the very elementary work of Jaḡmini (d. 1344), *al-Molaḵaṣ fi'l-hayʾa*, and its commentaries, especially the one authored by Qāżi-zāda Rumi (d. ca. 1440). But none of these later works could be regarded as serious works of astronomy on a par with the *Taḏkera* and its commentaries.

The last astronomical work of Ṭusi, which was probably produced posthumously, was of the *zij* type. A zij, a word apparently derived from Old Persian meaning something akin to a string or set of strings (Kennedy, 1956, p. 123), is an astronomical handbook which gives, among other things, a set of tables of mean motions and equations for the planets, all arranged in numerical tabular forms, and all preceded by a descriptive section that gives instructions on how to use those tables in order to extract the position of any planet for any time. Since mean motions of the planets were recorded in days, months, and years, a zij usually also contained a section on the various calendars that dealt with years and that were known in the Islamic civilization. This section of the zij would also contain its own tables of equations among the various calendars to allow the user to move the mean motion from one calendar to another. In those regards Ṭusi's zij was no exception. The purpose of all that activity, however, was primarily astrological, because the casting of horoscopes of any kind necessarily required the knowledge of the planetary positions for the time for which the horoscope was to be determined. Since Ṭusi had paid some attention to the field of astrology by writing an elementary work on the subject of determining the positions of the planets in thirty chapters, which was therefore referred to as *Si faṣl*, or more comprehensively as *moḵtaṣar dar maʿrefat-e taqwim*, it was natural that he would also want to pursue the topic more fully. In order to do that accurately, he had to update the mean motions of the planets by conducting fresh observations and use the new values in order to produce more precise predictions from a horoscope. But observatories were very costly then, as they are now, and one could not undertake such projects without enormous financial support.

The occasion for the production of a new zij presented itself when Ṭusi joined the court of Hülegü, after the fall of the Ismāʿili fortress of Alamut in 1256. Having had his life spared on account of his knowledge of astrology, a subject much valued by his new patron Hülegü, Ṭusi seized the opportunity and requested sufficient funding from his new patron to conduct fresh observations for a period of thirty years, with the valid pretext that he needed to observe the slowest known planet, which was then Saturn, whose cycle took thirty years to be completed. Fortunately for Ṭusi, Hülegü agreed to the request. After the fall of Baghdad in 1258, Ṭusi was allocated enough endowment funds for a new observatory. He was also allowed to have his pick of the books that could be salvaged from Baghdad and the other cities of Iraq during the Mongol devastation. He even had his pick of a librarian for his new observatory, a young man from Baghdad called Ebn al-Fowāṭi (d. 1323). Hülegü's generosity may seem somewhat puzzling. But one should remember that, after assuming the title of Il-khan, i.e. ruling on behalf of the Great Khan Möngke, he was merely allocating funds he had recently secured through conquest to his trusted vizier, Ṭusi, who had played such a pivotal part in achieving these conquests.

With the funding secured, Ṭusi gathered the most famous astronomers and engineers of his time and, before Hülegü had any chance to change his mind, quickly commenced the construction of what was later to become one of the most important observatories in Islamic civilization, in the city of Maragha (Marāḡa) in northwest Persia, which had become the Il-khanid capital. With the assistance of the most able engineer and astronomer of that time, the then famous Moʾayyad-al-Din al-ʿOrżi of Damascus (d. 1266), the observatory seems to have become operational by about 1261-62, i.e., within a few years from the commencement of its construction in 1259 (Sayılı, 1960, pp. 187-223). Fortunately for Ṭusi and his collaborators, the death of Hülegü in 1265 does not appear to have disrupted their activities at the observatory, and the patronage simply passed to Hülegü's son, Abaqa (d. 1282). By that time several others had joined the observatory community, most notably the astronomer Yaḥyā b. Abi'l-Šokr Maḡrebi (d. 1283), whose life had also been spared in a battle near Damascus on account of his knowledge of astrology. Among the distinguished names who had also joined the group at Maragha was the already mentioned and famous student of Ṭusi, Qoṭb-al-Din Širāzi. The major work that was produced by this group of luminaries was the magisterial zij known as the *Zij-e Ilḵāni*, which was of course dedicated to the ruling Il-Khan who funded the institution in the first place.

The people mentioned so far in association with the Maragha activities are the people whose works have survived and whose participation in the astronomical works of Maragha can be identified. There are others, like Najm-al-Din Dabirān of Qazvin, Faḵr al-Din Marāḡi of Mosul, and Faḵr-al-Din Ḵelāṭi of Tiflis, whose contribution to the project remains uncertain, although they are mentioned in the introduction of the *Zij-e Ilḵāni*, which was either produced at that observatory by Ṭusi himself or completed very shortly after his death by one of his associates. Unlike Maḡrebi, whose name, strangely enough, was not even mentioned by Ṭusi in the same introduction to the *Zij-e Ilḵāni*, but who has left us a detailed description of the observations he conducted at Maragha and the zij he had composed which was based on those observations (Saliba, 1983), none of the others has produced similar testimonials. Furthermore, the *Zij-e Ilḵāni* itself does not contain any information on the manner in which the new observations were conducted or how those observations were incorporated in the Zij.

What is certain is that the *Zij-e Ilḵāni* proved to be a great success. Written originally in Persian, it was quickly translated into Arabic as well as other languages. Arabic translations were indeed done, and the text itself was re-edited in Arabic more than once by scholars who were otherwise relatively unknown, like Ḥasan b. Ḥosayn Šāhanšāh Semnāni (c. 1393), Šehāb al-Din Ḥalabi (c. 1455), ʿAli b. al-Refāʿi al-Ḥosayni (c. 1527). In addition, there were commentaries and further re-workings of its contents by many others. One particular commentator on a previous commentary of this zij by the name of Maḥmudšāh Ḵolji should be singled out on account of the fact that long extracts of his work were picked up much later by John Greaves (d. 1652), the Oxford professor of astronomy, who took a great interest in Islamic astronomical works and who published his selection of Ḵolji in 1650 and then again in 1652 (Greaves, 1652; Toomer, p. 174). The geographical section of *Zij-e Ilḵāni*, which has lists of longitudes and latitudes of famous cities, together with the similar section from the *Zij-e Solṭāni* of Uluḡ Beg (d. 1449), had been the subject of a publication by Greaves in 1648.

Parts of the *Zij-e Ilḵāni* were also translated into Byzantine Greek

(Neugebauer, 1960, pp. 28, 31, 32, and passim; Mercier, 1984) during the 13th and 14th centuries by scholars like Gregory Chioniades (ca. 1320) and George Chrysococces (ca. 1350; Mercier, 1984; Pingree, 1964, p. 144; 1985, pp. 24, 159, 263, and passim). Together with material from other Persian and Arabic zijes, the translations managed to supplant the more ancient Greek equivalent sources during the later Byzantine times, apparently because the zijes were deemed more up-to-date.

Taken together, it should be easy to see why Ṭusi's works attracted such widespread attention among students of the mathematical sciences in the Islamic lands, as well as much later and further afield, in – 17th century England. They were comprehensive, and ranged from the elementary introductions to the most creative advanced theoretical works. When one considers the text of Ḵolji that was translated by John Greaves, it becomes immediately apparent that this particular text was chosen for its pedagogical value, as it extracted from the *Zij-e Ilḵāni* the basic definitions of technical terms, and contained what could be called a glossary of the astronomical concepts that were used by Ṭusi. Such works would have constituted ideal introductory material for students of astronomy, even at such a late date as seventeenth-century England. The *Taḏkera*, as we have seen, must have played a similar role for the more advanced students all over the Islamic world.

Ṭusi's contribution to the mathematical sciences was far more profound, however, and went beyond his scholarly output. A formidable man of science, apparently endowed with tremendous administrative abilities, and politically astute and adept at seizing his opportunity at the right time, Ṭusi managed to create what could be described as one of the most advanced institutes of higher astronomical research of its time: the institute at his Maragha observatory. It is true that the pretext for the establishment of the observatory was to conduct new observations for the purposes of the production of a zij such as the *Zij-e Ilḵāni*, and the *zij-e adwār al-anwār* of Yaḥya b. Abi'l-Šokr Maḡrebi, just mentioned. But the actual community that was gathered at Maragha produced much more than that. The chief engineer, Moʿayyad-al-Din al-ʿOrḍi, who built the instruments of the observatory did not even produce a zij. Instead he had already come to Maragha with a hayʾa text on theoretical planetary theories, simply called *Ketāb al-hayʾa* (Saliba, 2001), which, together with Ṭusi's *Taḏkera* (itself apparently composed as the observatory was being constructed), became the basis for elaborate research on cosmological and planetary theoretical questions that had little to do with actual more updated observations.

These highly theoretical cosmological texts spurred discussions on the foundations of science. Judging by the work of one of the major figures, Qoṭb-al-Din Širāzi, who had studied at Maragha, it was those discussions that constituted the bulk of his voluminous output. In brief, Maragha became the crucible for the production of the "new astronomy" (*al-hay'a al-jadida*), a fitting description of it employed a century later by Ebn al-Šāṭer of Damascus (d. 1375).

This theoretical contribution of the Maragha group proved to be much more influential than Tusi's writings in the formation of the later astronomical traditions both in the Islamic world and in Europe. Without the serious critique to which Greek astronomy was subjected at the time, and without the painstaking attempt at producing more systematic, alternative mathematical astronomies, the works of subsequent astronomers, like the already mentioned Ebn al-Šāṭer of Damascus or Šams-al-Din Ḵafri or even those of Copernicus, would not be as easily understood and explicable as they can be at present in light of the work that was done at Maragha. After all, it was the two mathematical theorems that were devised by the same people who were assembled at Maragha, namely, the Ṭusi Couple and the similar ʿOrḍi Lemma, which was used to remedy the defects of the Greek astronomical theory that pertained to the motion of the upper planets, that became part and parcel of the astronomical production of every serious astronomer afterwards, including Copernicus himself, who made use of both of those theorems in his own work.

More importantly, one should not underestimate the impact of this will to find a 'new astronomy' that would replace the astronomy of the ancient Greeks on the final collapse of the Aristotelian cosmological system. By allowing themselves to drive that system to its logical conclusions, those astronomers of Maragha may have unwittingly set the stage for the much deeper questioning of the system by someone like Ebn al-Šāṭer of Damascus, and may have indeed set the stage for its final overthrow during the European Renaissance. The fact that Copernicus could still make use of their technical mathematical production indicates the dynamic role that those Maragha astronomers played. It is a fitting tribute to Ṭusi that he could achieve all that in the relatively short span of about half a century, from the time when he came to study with Kamāl-al-Din of Mosul in the 1220s until his death in Baghdad in 1274.

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