



# MĀHĀNI, ABU 'ABD-ALLĀH MOḤAMMAD

**MĀHĀNI, ABU 'ABD-ALLĀH MOḤAMMAD** (ابوعبدالله محمد ماهانی) b. Isā, mathematician and astronomer who flourished in the second half of the 9th century (fl. ca. 246/860). His name indicates that either he or his ancestors were originally from Māhān, a town in the province of [Kerman](#). The only information we have regarding the period in which he lived comes from the Egyptian astronomer Abu'l-Ḥasan 'Alī b. Yunos (d. 399/1008), who describes in the *Ḥākemite Tables* a series of astronomical observations made by Māhāni that spanned thirteen years, from Ramaẓān 239/February 854 to Ẹu'l-qa'da 252/November 866 (Caussin de Perceval, 1804, pp. 99, 101-13, 164-66). According to Ebn al-Nadim, Māhāni was a learned arithmetician and geometer (*men 'olamā' al-aṣṣḥāb al-a'dād wa'l-mohandesin*); Qeṭṭi adds that his merit was generally recognized among his peers, and that he lived in Baghdad (Ebn al-Nadim, ed. Flügel, I, p. 271; Qeṭṭi, ed. Lippert and Müller, 1903, p. 284).

*Extant works.* Three works of Māhāni have come down to us: the treatise on the difficulty concerning ratio (*Resāla fi'l-moškel men amr al-nesba*); the commentary on the tenth book of Euclid's work (*Tafsir al-maqāla al-'āšera men ketāb Oqlides*); and the treatise on the knowledge of the azimuth at any time and in any place (*Maqāla fi ma'refat al-samt le ayya sā'a aradta wa fi ayya mawze' aradta*).

The Treatise on ratio. Euclid (Oqlides) had expounded in book V of the



*Elements* the theory of proportion applicable to every kind of magnitude. Definitions V.5 and V.7 provided criteria for the equality and inequality of ratios between magnitudes, which in each case involved a comparison of certain equimultiples of the magnitudes. These definitions, notably because of the difficulties involved in understanding them, gave rise to many commentaries. In some of these commentaries, Definitions V.5 and V.7 were replaced by alternate definitions, the so-called “anthyphairetic” definitions of equal and greater ratio, which had recourse to a process usually known as the Euclidean algorithm, but which some historians also call “anthyphairetic” after a Greek word meaning “reciprocal subtraction” (for more details see “Concepts of ratio and proportionality” in “[KHAYYAM, OMAR vi. As Mathematician](#)”). In his treatise on ratio, the first mathematical text in which the anthyphairetic definitions are explicitly mentioned, Māhānī proves that these definitions are in fact equivalent to the corresponding Euclidean definitions.

The treatise begins with a laconic introduction, in which Māhānī explains that he intends to prove Euclid’s Definitions V.5 and V.7, following a directive of Tābet b. Qorra (211-88/826-901), namely, that “the science of the ratio of magnitudes and their proportionality according to the rules is something one will be able to attend to by starting from the knowledge of ratio according to the numerical way, then from the propositions which are in the beginning of the tenth Book” (*Resāla fi’l-moṣkel*, ed. and tr. Vahabzadeh, 2002, p. 31). This is an allusion to Propositions X.2 and X.3 of the *Elements*, in which Euclid had used anthyphairetic as a criterion for the incommensurability of two magnitudes, and for finding the greatest common measure of two commensurable magnitudes.

In the following definitions, Māhānī explains that the ratio between every pair of magnitudes is a certain state that occurs to each magnitude when measured by the other, and characterizes this state by means of the anthyphairetic process; accordingly two ratios will be equal, if this process, when applied to each pair of magnitudes, unfolds in the same manner. He then defines greater ratio, also by means of anthyphairetic (*Resāla fi’l-moṣkel*, ed. and tr. Vahabzadeh, 2002, pp. 31-3).

Then follow three propositions, in which he proves that his definitions of equal and greater ratio are equivalent to Euclid’s Definitions V.5 and V.7, respectively; in the fourth and last proposition, he explains how to find the equimultiples mentioned in Definition V.7, assuming that one ratio is greater



than the other according to the anthyphairetic definition (*Resāla fi'l-moškel*, ed. and tr. Vahabzadeh, pp. 33-40).

The Commentary on book X of Euclid's *Elements*. Euclid had first introduced the concept of irrationality in book X of the *Elements*. The main purpose of the book was to distinguish two kinds of magnitudes, namely, those that are expressible and those that are irrational, and to provide a rigorous classification of thirteen species of irrational lines (Euclide d'Alexandrie, III, pp. 11, 51-52). After having defined the notions of commensurability and incommensurability, he distinguished two kinds of lines (the lines considered in book X are always finite straight lines). First he took an assigned line, which he called "expressible" ("rational" in Heath's tr.). He called "expressible" those lines that are either commensurable with the assigned line, or whose square is commensurable with the square of the assigned line, and called the other lines "irrational" (Euclide d'Alexandrie, III, pp. 25-42; Heath, III, p. 10). Then followed 115 propositions, most of which were devoted to the construction and distinction of the thirteen species of irrational lines (these are, in modern notation, of the form  $\sqrt{\sqrt{ab}}$ ,  $\sqrt{a \pm \sqrt{b}}$ ,  $\sqrt{\sqrt{a} \pm \sqrt{b}}$ ,  $\sqrt{a \pm \sqrt{b}}$ ,  $\sqrt{(\sqrt{a} \pm \sqrt{b})}$ , and  $\sqrt{(\sqrt{a} \pm \sqrt{b})}$ ), where  $a$  and  $b$  are commensurable with the assigned line and verify certain specific conditions.

Māhānī's aim in his commentary is to define the species of surd (i.e., irrational) lines and expressible lines, and to differentiate them. He begins by distinguishing between expressible lines and surds:

The lines when uttered have two meanings. Either they are expressible, as when we say 'ten,' 'twelve,' 'three-and-a-half,' 'six-one-third,' and likewise all the lines whose magnitude can be uttered and whose quantity can be expressed. Or they cannot be expressed and are called surds, that is, those lines whose magnitude cannot be uttered and whose quantity cannot be expressed; as the square roots of numbers that are not squares, for instance ten, fifteen, and twenty; and as the cube roots of numbers that are not cubes." (*Tafsir al-maqāla*, ed. and tr. Ben Miled, 2005, p. 287)

In general, Māhānī considers the  $n$ th-roots of rational numbers, and what is obtained by composition (*tarkib*) and separation (*tafṣil*; i.e., in modern terminology, by addition and subtraction).

Māhānī differs from Euclid in some important respects, notably by interpreting numerically the lines considered in book X, that is, as rational



numbers or as the roots of rational numbers. Besides this, he considers the lines corresponding to the square root of non-square numbers as surds, whereas Euclid considered them as expressible (e.g., for Māhānī  $\sqrt{2}$  is a surd; whereas for Euclid,  $\sqrt{2}$ , or rather the diagonal of a square whose side is the assigned line, is expressible, since its square is commensurable with the square of the assigned line (*Tafsir al-maqāla*, ed. and tr. Ben Miled, 2005, p. 287; Elucide d'Alexandrie, III, p. 39).

Māhānī then divides the surds according to genus (*men jehat al-jens*) into two species: plane (*baṣiṭ*) and solid (*jermi*). Plane surds are the square roots, the square roots of square roots, ... (i.e.,  $\sqrt{a}$ ,  $\sqrt{\sqrt{a}}$ , ...), and what is obtained by composition and separation (e.g.,  $\sqrt{a \pm b}$ ,  $\sqrt{\sqrt{a \pm b}}$ ,  $a \pm \sqrt{b}$ ,  $a \pm \sqrt{\sqrt{b}}$ , ...). Solid surd lines are the cube roots, the cube roots of square roots, the cube roots of cube roots, ... (i.e., the  $n$ th-roots of rational numbers where  $n = 3, 3.2, 3^2, \dots$ ), and what is obtained by composition and separation (*Tafsir al-maqāla*, ed. and tr. Ben Miled, 2005, p. 287).

Surds can also be divided according to quality (*men jehat al-ṣefat*) into two divisions: simple (*mofrad*) and compounded (*morakkab*). Simple surds are those consisting of one term, which is either plane or solid. Compounded surds are obtained by combining or by separating simples surds, either plane simple surds or solid simple surds (*Tafsir al-maqāla*, ed. and tr. Ben Miled, 2005, p. 289).

Māhānī has thus provided a general and exhaustive classification of all the species of surd lines. He then restricted his discussion to what Euclid had mentioned in book X and comment upon it “so that all the species mentioned by the scholar [i.e., Euclid] may be distinguished clearly by the measure of number and by calculation” (*Tafsir al-maqāla*, ed. and tr. Ben Miled, 2005, p. 289).

After reminding the reader of the definition of expressible lines (which, as already mentioned, does not correspond to Euclid's), Māhānī says that surds are either simple (*mofrad*) or non-simple. Simple surds are the square roots, the square roots of square roots, the square roots of square roots of square roots, ... (e.g.  $\sqrt{5}$ ,  $\sqrt{\sqrt{10}}$ ,  $\sqrt{\sqrt{\sqrt{8}}}$ , ...). Non-simple surds are divided into two divisions: joined (*mottaṣel*) and separated (*monfaṣel*). Joined surds are the junction (*etteṣāl*) of two surd lines, or the junction of a surd line and an expressible line. Separated surds are the separation (*enfeṣāl*) of a surd line from a surd line, or the separation of a surd line from an enunciable line, or



the separation of an enunciable line from a surd line (*Tafsir al-maqāla*, ed. and tr. Ben Miled, 2005, pp. 289-91).

He then divides joined lines into two divisions: simple (*baṣiṭ*) and compounded (*morakkab*). A simple junction is, for instance, the junction of two simple surds, or the junction of a simple surd with an expressible line. To this division belong the binomial and the bimedial straight lines mentioned in book X (i.e., lines of the form  $\sqrt{a}+\sqrt{b}$  and  $\sqrt{\sqrt{a}+\sqrt{b}}$ , respectively; Matvievskaia, pp. 255, 260). In the only extant manuscript, the treatise stops in the middle of a sentence and is followed by the words “it is until here that this discourse was found” (*Tafsir al-maqāla*, ed. and tr. Ben Miled, 2005, p. 291).

The Treatise on the knowledge of the azimuth. We do not know of any edition or translation of this work; an analysis of its content in German is found in Paul Luckey’s paper on Islamic mathematics (Luckey, 1948, pp. 500-03; summary in Sezgin, *GAS* V, pp. 260-61, and VI, pp. 155-56). In this work Māhānī determines the azimuth of the sun in terms of its declination, its altitude, and the latitude of the given locality (Rosenfeld, p. 306; Rosenfeld and Youschkevitch, p. 155).

*Lost Works.* Ebn al-Nadīm and Qeṭī mention, besides Māhānī’s treatise on ratio, a treatise on the latitudes of stars (*Resāla fī ‘oruḍ al-kawākeb*) and a treatise on twenty-six propositions of the first book of Euclid that do not require at all a *reductio ad absurdum* (*Fi setta wa ‘ešrin šaklan men al-maqālat al-awlā men Oqlides allati lā yoḥtāj fī šay’ menhā ela’l-kolḥ*), both of which are lost; they also mention a commentary (*šarḥ*) on the fifth book of Euclid’s *Elements*, which may possibly be the same as his treatise (*resāla*) on ratio (Ebn al-Nadīm, ed. Flügel, I, pp. 266, 271; Qeṭī, ed. Lippert and Müller, pp. 64, 284.)

Ebrāhīm b. Senān b. Tābet (296-335/909-46) tells us in his autobiography (*Resāla fī waṣf al-ma‘āni allati estakrajahā fī’l-handasa wa ‘elm al-nojum*) that Māhānī wrote a treatise on the measurement of the parabola, which consisted of preliminary arithmetical lemmas followed by five (or six) propositions proven by *reductio ad absurdum* (Bellosta and Rashed, pp. 18-19). He also alludes in his treatise on the instruments of shadows (*Fi ālāt al-aḏlāl*) to a work in which Māhānī had determined the ascendent by means of the sundial (Sesiano, p. 406; Bellosta and Rashed, pp. 340-41).

According to an anonymous tract, possibly written by the Persian mathematician and astronomer [Abu Sahl Vijan Kuhi](#) (10th cent.), Māhānī



wrote a commentary on Book II of Archimedes' treatise *On the Sphere and Cylinder*. Māhānī, we are told, solved eight of the nine propositions of this book; but he failed to provide a perfect solution of Proposition 4, because of a difficult lemma that was needed in its solution. He then tried to solve it by algebra, and reduced the problem to the resolution of a cubic equation (Ḳayyām, pp. 96, 101-02). Khayyām states that it was Māhānī's idea to try to solve this lemma by means of algebra and that he was led to a cubic equation which he was unable to solve despite all his efforts (Ḳayyām, p. 2).

According to the 10th-century astronomer Aḥmad b. Abi Sa'īd Heravi, a group of geometers had recourse to Māhānī in order to emend the *Sphaerica* of Menelaus (fl. ca. 100). Māhānī was able to emend the first book and some propositions of the second; but he stopped at the tenth proposition of the second book because of its difficulty and did not go beyond it (Krause, 1936a, pp. 25-32.)

Edward B. Plooi (1997, p. 4) mentions among Māhānī's works "an explanation of obscure places in Book 13 [of the *Elements*]." All sources referred to by him (i.e., Brockelmann, *GAL S I*, p. 383; Sarton, 1927, pp. 597-98; Suter, 1900, pp. 26-27; Kapp, 1935, pp. 60-61; Krause, p. 450) were checked, but no mention of this text could be found; Fuat Sezgin does not mention it either.

## BIBLIOGRAPHY

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A list of the works of Māhānī and the extant manuscripts thereof is provided in Sezgin, *GAS V*, pp. 260-62; VI, pp. 155-56; and VII, p. 404. To the seven MSS of the *Resāla fi'l-nesba* mentioned by Sezgin, one must add Qom, Ayatollah Mar'aṣī, MS 4521, and Tehran, Sepahsālār, MS 690 (fols. 175-82); besides these, MS Berlin 6009 is now in the Jagiellonian Library in Cracow under the reference MS Orient. Fol. 258.

Editions and translations.

*Resāla fi'l-moṣkel men amr al-nesba*, ed. and Eng. tr. Bijan Vahabzadeh, in idem, "Al-Māhānī's Commentary on the Concept of Ratio," *Arabic Sciences and Philosophy* 12/1, 2002, pp. 9-52, esp. pp. 31-52; Fr. tr. in idem, "Le commentaire



d'al-Māhānī sur le concept de rapport,” available online at <https://hal.archives-ouvertes.fr/hal-01185287> (submitted 1 September 2015).

*Tafsir al-maqāla al-āšera men ketāb Oqlides*, ed. and tr., Marouane Ben Miled, in idem, “Les commentaires d'al-Māhānī et d'un anonyme du Livre X des *Eléments* d'Euclide,” *Arabic Sciences and Philosophy* 9, 1999, pp. 89-156, esp. pp. 122-44 and 142-43; and in idem, *Opérer sur le continu: Traditions arabes du Livre X des Eléments d'Euclide*, Carthage, 2005, pp. 286-92.

Other primary sources.

Armand-Pierre Caussin de Perceval, ed. and tr., “Le livre de la grande Table hakémite,” in *Notices et extraits des manuscrits de la Bibliothèque nationale et autres bibliothèques* 7, 1804, pp. 16-240.

Ebn al-Nadim, ed. Flügel, 2 vols., Leipzig, 1871-72; repr., ed. Fuat Sezgin, *Series Historiography and Classification of Science in Islam* V, two vols., Frankfurt am Main, 2005.

Euclide d'Alexandrie, *Les Eléments*, tr. Bernard Vitrac with commentary, General Introd. by Maurice Caveing, 4 vols., Paris, 1990-2001.

Thomas L. Heath, *The Thirteen Books of Euclid's Elements*, tr. from the text of Heiberg, with introd. and comm., 2nd ed., 3 vols., Cambridge, 1926; repr., New York, 1956.

'Omar Ḳayyām, *Resāla fi'l-barāhin 'alā masā'el al-jabr wa'l-moqābala*, ed. and tr. Franz Woepcke, as *L'Algèbre d'Omar Alkhayyāmī*, Paris, 1851; repr. in *Islamic Mathematics and Astronomy* 45, Frankfurt am Main, 1998, pp. 1-206.

'Ali b. Yusof Qefti, *Ta'riḳ al-ḥokamā'*, ed. Julius Lippert and August Müller, Leipzig, 1903; repr., *Islamic Philosophy* 2, Frankfurt am Main, 1999.

Secondary sources and studies.

Hélène Bellosta and Roshdi Rashed, *Ibrāhīm Ibn Sinān: Logique et géométrie au X<sup>e</sup> siècle*, *Islamic Philosophy, Theology and Science: Texts and Studies*, XLII, Leiden, Boston, and Köln, 2000.

Carl Brockelmann, *Geschichte der arabischen Litteratur: Supplement*, 3 vols., Leiden, 1937-42.



Yvonne Dold-Samplonius, "Al-Māhānī," in Helaine Selin, ed., *Encyclopaedia of the History of Science, Technology, and Medicine in Non-Western Cultures*, 2nd ed., Berlin, Heidelberg, and New York, 2008, pp. 141-42.

Jan P. Hogendijk, "Anthyphairetic Ratio Theory in Medieval Islamic Mathematics," in Yvonne Dold-Samplonius, Joseph W. Dauben, Menso Folkerts, and Benno Van Dalen, eds., *From China to Paris: 2000 Years Transmission of Mathematical Ideas*, Stuttgart, 2002, pp. 187-202 (contains an edited version and Eng. tr. of some passages of the *Resāla fī'l-moškel men amr al-nesba*).

A. G. Kapp, "Arabische Übersetzer und Kommentatoren Euklids, sowie deren math.-naturwiss: Werke auf Grund des Ta'rīkh al-Ḥukamā' des Ibn al-Qiftī: III," *Isis* 24, no. 1, 1935, pp. 37-79.

Max Krause, *Die Sphärik von Menelaos aus Alexandrien in der Verbesserung von Abū Naṣr Maṣṣūr b. 'Alī b. 'Irāq: Mit Untersuchungen zur Geschichte des Textes bei den islamischen Mathematikern*, Abhandlungen der Gesellschaft der Wissenschaften zu Göttingen philologisch-historische Klasse 17, Berlin, 1936a.

Idem, "Stambuler Handschriften islamischer Mathematiker," in Otto Neugebauer, J. Stenzel, and O. Toeplitz, eds., *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*, Abt. B: Studien, 3, 1936b, Berlin, pp. 437-532.

Paul Luckey, "Beiträge zur Erforschung der islamischen Mathematik," *Orientalia*, N.S., 17/4, 1948, pp. 490-510.

Galina Matvievskaia, "The Theory of Quadratic Irrationals in Medieval Oriental Mathematics," in David A. King and George Saliba, eds., *From Deferent to Equant: A Volume of Studies in the History of Science in the Ancient and Medieval Near East in Honor of E. S. Kennedy*, New York, 1987, pp. 253-77 (contains an Eng. tr. of some passages of the *Tafsir al-maqāla al-'āšera men ketāb Oqlides*).

Edward Bernard Plooi, *Euclid's Conception of Ratio and His Definition of Proportional Magnitudes as Criticized by Arabian Commentators*, Rotterdam, 1950; repr. in *Islamic Mathematics and Astronomy* 19, Frankfurt am Main, 1997, pp. 167-243 (contains an Eng. tr. of the first definitions of the *Resāla fī'l-moškel men amr al-nesba*).

Boris A. Rosenfeld, "Geometric Trigonometry' in Treatises of al-Khwārizmī, al-



Māhānī and Ibn al-Haytham,” in Menso Folkerts and Jan P. Hogendijk, eds., *Vestigia Mathematica: Studies in Medieval and Early Modern Mathematics in Honor of H. L. L. Busard*, Amsterdam and Atlanta, Ga., 1993, pp. 305-8.

Boris A. Rosenfeld and Adolf P. Youschkevitch, revised and enlarged by Hélène Bellosta, “Géométrie,” in Roshdi Rashed and Régis Morelon, eds., *Histoire des sciences arabes II: Mathématiques et physique*, Paris, 1997, pp. 121-62.

George Sarton, *Introduction to the History of Science I: From Homer to Omar Khayyam*, Baltimore, 1927.

Jacques Sesiano, “Muḥammad b. Īsā b. Aḥmad al-Māhānī,” in *EI*<sup>2</sup> VII, 1993, p. 406.

Heinrich Suter, *Die Mathematiker und Astronomen der Araber und ihre Werke*, Leipzig, 1900; repr., New York and London, 1972.

Bijan Vahabzadeh, “Khayyam, Omar vi: As Mathematician,” *EIr.*, online edition, available at <https://www.iranicaonline.org/articles/khayyam-omar-vi-mathematician> (accessed on 7 May 2014).

Bernard Vitrac, “Omar Khayyām et l’anthyphérèse: Étude du deuxième Livre de son commentaire sur certaines prémisses problématiques du livre d’Euclide,” *Farhang* 14, Tehran, 2002, pp. 137-92; available online at <https://hal.archives-ouvertes.fr/hal-00174930> (section IV.C.11 of this paper contains a remarkable conjecture on why Tābet b. Qorra directed Māhānī to characterize ratio by means of anthypharesis).