



KĀŠĪ, ĠĪĀṬ-AL-DIN

KĀŠĪ [or Kāšāni], **ĠĪĀṬ-AL-DIN** JAMŠĪD b. MAS‘UD b. MOḤAMMAD (b. ca. 787/1386, d. ca. 832/1429), mathematician, astronomer and scientific instrument-maker of the highest rank. He was also known as a practitioner of medicine, being sometimes referred to as “al-Ṭabīb” (the physician), and he mentioned in one of his letters to his father (see below) that he dabbled also in music.

LIFE

The details of Kāšī’s personal life are not known with much certainty. We can only surmise that he was born in Kashan on the basis of the *nesba* Kāšāni [or Kāšī] that is usually provided as part of his name. According to Edward S. Kennedy, who devoted several studies to Kāšī, the earliest date that is specifically referred to in his writings is the date of an astronomical observation that he conducted of the lunar eclipse which took place on 12 Ḍu’l-Ḥejja, 808/2 June 1406 (Kennedy, 1960, p. 1). If it is assumed that this brilliant astronomer and mathematician started observing early in his adult life, at around the age of twenty, then it may be estimated that he was probably born around the year 787/1386, some six to seven years before the birth of his main patron, Oloḡ Beg (795-852/1393-1449), the grandson of Timur (Tamerlane).

Kāšī’s death date is equally problematic. Heinrich Suter states that he died around 839/1436 (pp. 173-74) without identifying the source of this date, while Kennedy cites certain notes on the title pages of at least two manuscripts of an astronomical work by Kāšī, both of which stipulate that his death took place in



832/1429 (Kennedy, 1960, p. 7). The latter date is much more likely, since the *zij* (astronomical table) of Oloḡ Beg which was completed around the year 843/1440, some nine years before the death of Oloḡ Beg himself, refers to Kāši as having already died before completing the observations at the Samarqand observatory upon which the *zij* was to be based, and that he was succeeded in that task by Q“āzizāda Rumi (d. 839/1436), who had also died before its completion, leaving the work to be finished by a younger student of Oloḡ Beg by the name of ‘Alā’-al-Din Qošji. With two astronomers succeeding Kāši in the completion of the *zij*, the most likely implication is that a significant period must have separated the death of Kāši and the eventual completion of the work by Qošji in around 1440. Therefore the death date of 1429 fits much better than 1436 with this information, while also corroborating the notes that have been cited by Kennedy.

Kāši’s travels in search of a patron seem to have taken him from Kashan to Isfahan and Samarqand, among other places, before he finally found a secure position under the admiring patronage of Oloḡ Beg. Oloḡ Beg spoke of him in the introduction of his own *zij* (See Sedillot, ed. and tr., p. 289; French translation, p. 5) in such glowing terms as “the pride of scientists in the world, the one who completed the sciences of the ancients, and resolved the most difficult of their problems.” Coming from such a distinguished scientist as Oloḡ Beg himself, this praise is not without significance. That Kāši should have been invited to Samarqand by Oloḡ Beg in order to participate in the construction of a new observatory that was planned by this enlightened patron, and eventually to direct its activities, further asserts his reputation among his own contemporaries for excellence in the astronomical sciences, and it may also provide a clue for determining the time when he finally arrived in Samarqand. The observatory was founded in the year 823/1420, but construction probably did not begin until around the year 825/1422 (Sayılı, 1960, p. 271 ff.). Since Kāši signals in a letter to his father that construction was already in process by the time he had arrived in Samarqand, and that he had advised those responsible that some of the instruments that had already been constructed needed to be dismantled, this would mean that he probably reached Samarqand around the year 825/1422.

Kāši’s own assessment of the construction of the Samarqand observatory and his rank at the court of Oloḡ Beg are available in the form of two letters that he had sent to his father, describing the scientific work at the intellectually thriving city of Samarqand (Sayılı, Bagheri, Kennedy, Ṭabāṭabā’i). They reveal



that Oloğ Beg's construction of an advanced school of learning, the structure of which is still standing as an architectural monument, had preceded that of the observatory, and that some sixty distinguished scientists, particularly astronomers, were already assembled in it by the time Kāši appeared on the scene. Among those scientists was the famous teacher and astronomer Q"āzizāda Rumi, mentioned above, of whom Kāši speaks with high praise as "the most learned among them" (i.e. the scientists at Samarqand), before proceeding to describe him condescendingly in the same letter as "only a beginner in theoretical astronomy." (Sayılı, 1985, p. 107).

The value of these letters, which must date from the years 828/1424-25 (i.e. some five years before Kāši's death and while the observatory was still being constructed), lies primarily in the fact that they have preserved for us a glimpse of the scientific activity at Samarqand during the reign of Oloğ Beg, when scientific planning and group work was at its best. We sense from the preserved letters not only the competitiveness between the scientists themselves, but also some of the kinds of difficulties those scientists were facing in their attempts to erect an observatory that could at least match, if not surpass, the one that had been built some two centuries earlier at the city of Marāğa, by the equally famous pair of astronomers Mo'ayyad-al-Din al-'Orđi (d. 664/1266) and *Naşir-al-Din* ʦusi (d. 672/1274).

AS MATHEMATICIAN AND ASTRONOMER

Kāši recounts the difficulties the engineers at Samarqand faced in erecting observational instruments that were large enough to ensure accuracy and yet not so large that the kind of material used might be in danger of warping. Kāši resolved such practical details, as well as solving more advanced theoretical questions, one after the other, to the amazement of the assembled scientists. While allowing for some exaggeration on the part of Kāši, as he seems to have intended to stir the pride of his father in his achievements, one cannot escape the impression of a distinguished scientist at work, surrounded by much less qualified colleagues. For instance, we are told that, although theoretical instruction at the school was conducted on a daily basis, and Oloğ Beg himself would not only attend the classes that took place there but would also sometimes lecture on astronomical subjects himself, nonetheless Kāši would only attend in the company of Oloğ Beg, and only to give advice, resolve difficult problems, and learn from the discussions anything he did not already



know. In fact, the vibrant activity at the school and the equally vibrant competition that took place there have also been recorded; Mollā Faḥ-Allāh Širvāni (d. 891/1486), one of the students who attended those classes, went on to write a distinguished work on astronomy in which he describes his encounters with Oloḡ Beg and his entourage during a class session that was chaired by Qāzizāda (Širvāni, *Šarḥ al-Taḍkera*, fols. 14v-15r, seq, 108v-109r).

It was in such an environment that Kāši finally completed his two most monumental works, on astronomy and mathematics, respectively. He had apparently started his major astronomical work, the *Zij-e k̄āqāni*, some ten years before coming to Samarqand, but was unable to finish it until he finally gained the patronage of Oloḡ Beg, to whom the completed work was dedicated. The second book, his *Meftāḥ al-Ḥesāb* [or *al-Ḥossāb*] (ed. Nābolosi, 1977), on mathematics, is discussed below.

Although the text of the *Zij-e k̄āqāni* has not been published yet, its contents have been described in detail in Kennedy's classic study (1956, pp. 164-66). Since it was constructed as an update of the *Zij-e ilkāni* which had been composed by Ṭusi some two hundred years earlier, it followed closely the format of the latter and thus was similar in format to the roughly 200 *zijas* that had been written in the Islamic tradition between the 3rd/9th and 9th/15th centuries. By Kāši's time there was an established tradition that determined what such works must contain. The author's individual brilliance can nonetheless be observed in the details. For instance, some astronomers may find new tables based on fresh observations, others a more precise calculation, or further details about places, calendars, astrological functions, etc., that other *zijas* had not included.

Kāši's primary aim seems to have been to update the work of Ṭusi, which had obviously become obsolete some two hundred years after its composition. Nonetheless he included separate short treatises in which his mastery of the science was clearly displayed. One example is the determination of a higher approximation of the irrational number Pi which designates the ratio of the circumference of any circle to its diameter. In order to develop the crude approximations that had been used by earlier astronomers from Ptolemy's (fl. 150 AD) time onward to a higher degree of precision, Kāši composed a treatise, called *al-Resāla al-moḥiṭiya* (see below), specifically to determine the parameters of successively inscribed and subscribed polygons inside and around a circle, in order thus to calculate the value of Pi to sixteen decimal fractional units; his results remain accurate according to modern computer



calculations.

In a similar fashion, Kāšī's need to determine a more accurate trigonometric value of the sine of 1° led him to compose the short treatise called *Resālat al-watar wa'l-jayb* (see below), in which he managed again to create a fresh method of calculation, this time to enable him to compute the value of the sine of 1° to a decimal expansion of seventeen digits, all of which remain accurate according to modern standards. The details of both calculations are briefly reviewed in the thorough article on Kāšī by A. P. Youschkevitch and B. A. Rosenfeld (1973, pp. 255-62), which also provides an extensive bibliography.

In other instances, and in the context of writing his second monumental work, namely, his mathematical work *Meftāḥ al-Ḥesāb* [or *al-Ḥossāb*], he can be witnessed introducing new methods of calculation that were either unknown before his time or had been only briefly mentioned but not fully exploited. In the case of the general extraction of roots of numbers, of which the square root is a special case, Kāšī introduces an approximation technique that anticipated the method of Ruffii-Horner (formore information on this method of approximation and its role in Arabic mathematics, see Roshdi Rashed and Régis Morelon, eds., pp. 363, 367, 383-84, etc.)by some three hundred years and may have had a connection with a method that had been developed in Chinese mathematics a hundred years earlier but without becoming fully integrated as in Kāšī's work. Similarly, in the same workthere is a full articulation of the concept of decimal fractions that was, in a sense, a culmination of earlier developments in the Islamic tradition since the fourth/tenth century, and in the same vein anticipated the developments that took place in Europe in the seventeenth century.

Chronologically, the *Meftāḥ* seems to have been one of Kāšī's later works as it was completed in 830/1427. Its introduction contains a list of the author's other works, but this is definitely not a comprehensive list. The fact is that there are other works without doubt composed by Kāšī that are not included in this list, which refers only to the following: (1) *Zij-e kāqāni*, referred to above, (2) *Zij al-taṣḥilāt*, apparently lost, (3) *Sollam al-samā'*, dealing with the calculations of the distances of celestial bodies, (4) *Al-Resāla al-moḥiṭiya*, referred to above, (5) *Resālat al-watar wa-l-jayb*, referred to above, and (6) *Nozhat al-ḥadā'iq*, a treatise devoted to the description of an instrument that was invented by Kāšī with which one could calculate the positions of the planets by determining their longitude and latitude as well as their distance from earth, as he stipulates in the same introduction. This last treatise has been published by E.



S. Kennedy in a facsimile edition together with a translation and commentary (Kennedy, 1960).

Works by Kāšī that have not been included among those listed in the introduction of the *Meftāḥ* include: (7) *Meftāḥ al-asbāb fi 'elm al-zij*, known to exist in Mosul but not yet studied, (8) *Resāla dar sākṭ-i aštorlāb*, listed in Mašhad but also not yet studied, (9) *Resāla fi ma'refat samt al-qebila men dā'era hendiya*, also listed in Mašhad and again not yet studied, (10) *Resālat 'amal al-żarb be'l-taḳt wa'l-torāb*, published only in a lithograph edition and not yet studied, (11) *al-Resāla al-eqlilāmena (sic)* listed by Kennedy (1960, p. 7) as being mentioned by Kāšī himself, (12) *Moḳtašar dar 'elm hay'a*, not yet studied, (13) *Resāla dar šarḥ ālāt-e rašd*, already published in facsimile with translation and commentary by E. S. Kennedy (1961), (14) *Elḥāqāt al-nozha*, supplement to number 6 mentioned above, (15) *Talkiṣ al-meftāḥ*, a summary of the *Meftāḥ*, mentioned above, (16) *Ta'rib al-zij*, an Arabic translation of the introduction of Oloḡ Beg's *zij*, which may not have been composed by Kāšī even though it is attributed to him, and (17) *Natā'ej al-ḥaqā'eq* which is also attributed to him, but not yet studied properly.

When taken into consideration along with the *Meftāḥ* itself and the two letters that Kāšī wrote to his father, the titles of these works alone can confirm beyond any doubt Kāšī's reputation as a practicing astronomer, mathematician, and scientific instrument-maker. However, these works, or at least what has been discovered about them so far, remain silent about a different field of Islamic astronomy, in which Kāšī would therefore seem to have not participated effectively. The field in question dealt with the treatment of planetary theories in a cosmological mathematical context, and was very well represented by such works as 'Ordi's *Hay'a*, Ṭusi's *Tadkera*, and both Širāzi's *Toḥfa* and his *Nehāya* as well as Ebn al-Šāṭer's *Nehāyat al-sul fi tašḥiḥ al-uṣul*. Ṭusi's *Tadkera* alone attracted several commentaries, some of which we know were used as textbooks in Oloḡ Beg's school in classes chaired by the same Qāzizāda for whom Kāšī did not have much respect. In fact, one of Kāšī's letters mentions that even Oloḡ Beg himself would lecture on two of these books, the *Tadkera* itself as well as the *Toḥfa* by Širāzi (Sayılı, 1985, p. 95). The work of Faṭḥ-Allāh Širwāni also informs us that Qāzizāda used to lecture on the commentary of the *Tadkera* which was written by Neẓām-al-Din Nišāburi, one of Širāzi's students, and that such classes were of great interest to Oloḡ Beg himself (Širwāni, *Šarḥ al-Tadkera*, fols. 14v-15r ff., 108v-109r). Furthermore, Kāšī's colleague and subsequent successor both at the



observatory and in the completion of Oloḡ Beg's *zij*, the aforementioned Qoṣji, also wrote a short treatise in the same vein in which he confronted the problem of the motion of the planet Mercury from the perspective of *hay'a* (cosmography). Kāši himself brags in the same letter to his father that he had detected a mistake which was repeated in Širāzi's *Toḥfa* and the *Nehāya* as well as in Nišāburi's commentary on Ṭusi's *Taḍkera* and Jurāni's other commentary on the same *Taḍkera*, and that he had confounded the audience by his brilliance in this matter when he demonstrated that all those authorities had committed this same mistake. However, since Kāši did not care to specify in his letter to his father, we are left in the dark as to what kind of contribution he was claiming to have made to this particular field of astronomy.

The only other work by Kāši that may shed light on his competence in this type of research is the work listed as number 12 above, with the title *Moḳtaṣar dar 'elm-e hay'at*, which unfortunately remains unpublished, making it impossible to form a conclusive opinion on the matter. However, the fact that this work was written in Persian, and during the early part of Kāši's research career — in 812/1410-11 according to Youschkevitch and Rosenfeld — does not sound promising, because the most important and creative works that were written and studied in this field of astronomy during Kāši's lifetime, as already illustrated, were all written in Arabic, the language in which they continued to be written in for centuries to follow. Therefore, in view of the lack of studies on this specific point, it can only be surmised that Kāši's contribution in this field did not extend beyond pointing out the occasional mistake in other people's works, of which there were plenty. It is almost certain, however, that he did not undertake a comprehensive and formal treatment of planetary theories comparable with his work on the *zij* or his *meftāḥ*.

Kāši's works reveal a man with the temperament of a practical astronomer who was interested in the minute details of precision, whether it was the precision of scientific instruments or that of calculation techniques. However, he did not express much interest in cosmological questions, nor for that matter in the philosophical issues that had motivated those cosmological questions in the first place.



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