Published: January 1, 2000
Last Updated: March 8, 2013

BĪRŪNĪ, ABŪ RAYḤĀN III. MATHEMATICS AND ASTRONOMY

BĪRŪNĪ, ABŪ RAYḤĀN

iii. Mathematics and Astronomy

Ninety-five of 146 books known to have been written by Bīrūnī, about 65 percent, were devoted to astronomy, mathematics, and related subjects like mathematical geography (Kennedy, p. 152). The mathematical portions of his works were invariably devoted to applied, rather than theoretical, mathematics; nevertheless, in the process of solving problems, Bīrūnī did sometimes indulge in theoretical discussions. Similarly, although his main concern in astronomy was for computations, he also devoted attention to theoretical problems. The following assessment of Bīrūnī's contributions is based on his work in applied mathematics and on the theoretical portions of his astronomical works.

Theoretical concepts. Bīrūnī's major contribution to astronomy is *al-Qānūn al-masʿūdī fi'l-hay'a wa'l-nojūm* (Masʿudic canon of astronomy), covering the same ground as Ptolemy's *Almagest* but introducing new material. Most of Bīrūnī's original theoretical concepts are to be found in this work. Like the *Almagest*, the *Qānūn* contains theoretical derivations of astronomical parameters, as well as tabular functions to facilitate the computation of planetary positions. It thus differs from the works of most of Bīrūnī's predecessors and contemporaries who were concerned only with constructing astronomical tables (*zīj*) suitable for computation of planetary positions, usually without any discussion of the derivation of the parameters upon which the tables were based.

Although Bīrūnī did not write texts on algebra or geometry and his arithmetical works have not survived, he did introduce new mathematical concepts. For instance, in the *Qānūn* (bk. 3), in the course of a discussion devoted to the trigonometric functions used in astronomy, he defined the irrational number *pi* as the result of division of two other numbers (the circumference of a circle and the diameter), whereas his predecessors, including the Greek authors, had defined it as a geometric ratio. Elsewhere (bk. 6, chap. 8) he described the variation in the motion of the sun with respect to the earthly observer in mathematical language that modern historians of science have construed as among the earliest references to mathematical functional relationships (cf. Hartner and Schramm). In determining the mobility of the solar apogee, Bīrūnī followed his Muslim predecessors in departing from the traditional Greek astronomy of Ptolemy, but by means of more refined observational techniques he was able to go farther and to discover that the apogee has a motion of its own, distinct from the motion of precession.

In trigonometry his major contributions are to be found in *Ketāb maqālīd ʿelm al-hayʾa* (compendium on astronomy), in which he concentrated mainly on the applications of spherical trigonometry in astronomy and provided a detailed classification of spherical triangles and their solutions; in *Ketāb fī efrād al-maqāl fī amr al-ẓelāl* (exhaustive treatise on shadows), in which he developed the familiar trigonometric definitions further and applied them to such religious practices as determining times of prayer and finding the direction of Mecca; and in the third book of the *Qānūn*, in which he propounded trigonometric theorems equivalent to those related to the sums and differences of angles. It was in this last context that he developed his solution to the algebraic equation of the third degree (see below) as part of an attempt to compute the sine of 1°; the iteration method used in this calculation is no less sophisticated than methods developed by theoretical mathematicians. Furthermore, in these works, Bīrūnī not only defined all the trigonometric functions used today but also discussed methods of computing them from a circle with radius R = 1 (still used for this purpose); he also applied fully developed methods of second-order interpolation to computation of the intermediary values of these functions, thus demonstrating a clear understanding of functional relationships.

Elsewhere in the *Qānūn* (bk. 6, chap. 10; bk. 7, chap. 8) Bīrūnī showed similar sophistication in handling functional relationships by manipulating the equations of the sun and the moon so that the functions would always be positive; in contrast these relations varied between positive and negative in Ptolemy's (fl. 150) *Almagest* and *Handy Tables*. Bīrūnī also calculated the side of a nonagon, a problem resulting from his attempt to trisect an angle in order to compute the value of the sine of 1° (*Qānūn*, bk. 3, chap. 3); his calculations yielded the third-degree equation $1 + 3x = x^3$. He then solved the equation by inspection: root x = 1;50,45,47,13 (i.e. 1.846051929), which is correct to the third sexagesimal fraction (i.e. 1.84605). Theoretical considerations of this kind apparently stirred Bīrūnī's imagination, for he composed a book on the extraction of roots, unfortunately not extant. The most important aspect of this work, however, lay in Bīrūnī's ability to go beyond the strictly geometric approach of the Greeks to tackle the problem of trisecting an angle and in his recognition that algebraic solutions have the desired precision.

Applied mathematics. In mathematical geography Bīrūnī developed a new technique for measuring the difference in longitude between two given cities: He computed the longitudinal difference between Baghdad and Ḡazna at 24;20°, differing from the modern value by only eighteen minutes. In the same vein he described a method for calculating the circumference of the earth different from those preserved in Greek sources, though it may have been invented during the caliphate of al-Maʾmūn (198-201/813-17).

In the domain of numerical analysis and approximative techniques, Bīrūnī's ability to conceptualize in functional terms is equally clear. His calculation of the sine and tangent functions and their tabulation in the *Qānūn* (bk. 3, chap. 7-8) required him to develop an interpolation scheme involving second-order differences, for he was aware of the failure of a simple linear interpolation to account for extreme variation in functional values. It is curious that Bīrūnī did not adopt the similar, though not identical, method developed by Brahmagupta (b. ca. 598) in the *Khaṇḍakhādyaka*; he must have been aware of it, for he quoted from Brahmagupta's book in his own works several times (see *Qānūn*, ed. Hyderabad, p. 175 and passim). That he understood the power of his own second-order method of interpolation is apparent from his comment that it could be applied to all other tables.

Bīrūnī's attempts to record and classify all previously known methods for astrolabe projections, as well as methods that he himself proposed, in his comprehensive book on the astrolabe (*Ketāb fī estīʿāb al-wojūh al-momkena fī ṣanʿat al-aṣṭorlāb*) can perhaps also be included in the domain of applied mathematics. The problem of projections as such must have engaged his imagination, for he included some geographical map projections in another of his works *Maqāla fī-tasṭīḥ al-ṣowar wa tabṭīḥ al-kowar*, ed. A. S. Saʿīdān, *Derāsāt* 4, 1977, pp. 7-22). In all his writings Bīrūnī called attention to original concepts, though usually only in passing. In *Ketāb fī estīʿāb*, for instance, in discussing an astrolabe invented by his contemporary Abū Saʿīd Aḥmad Sejzī (ca. 339-415/951-1024), who had assumed for the purpose that the apparent daily rotation of the celestial spheres resulted from the motion of the earth, rather than of the celestial spheres themselves, he commented briefly that, though the motion of the earth is quite possible, the problem was one for natural philosophers, rather than for mathematicians, among whom he counted himself.

Bīrūnī does not appear to have been interested in the genre of astronomical writing in which Ptolemaic planetary models were considered as describing both the apparent motion of the planets and the physical spheres responsible for the kinematic forces acting upon them; this genre included Ptolemy's own *Planetary Hypotheses* and a number of later works usually containing the word *hayʾa* in their titles. The main concern of the authors was either to explain the Ptolemaic models, and thus to explain planetary motion as resulting from the motion of physical spheres, or to suggest new models for resolving the apparent contradiction between the physical and mathematical assumptions underlying the Ptolemaic models. Recent studies of these works are revolutionizing modern understanding of the role of Islamic astronomy in what later came to be known as Copernican astronomy, for the development of non-Ptolemaic models can be viewed as forerunners to Copernicus' own work. (E. S. Kennedy and I. Ghanem, eds., *The Life and Work of Ibn al-Shatir*, Aleppo, 1976, contains a collection of recent, pre-1976, studies dealing with planetary theories; D. King and G. Saliba, eds., *From Deferent to Equant*, Annals of The New York Academy of Sciences 500, 1987, p. xxvi, lists six recent works by G. Saliba.)

Bīrūnī's lack of concern with philosophical matters is apparent in his treatment of Sejzī's assumption about the earth's motion. In addition, he seems to have been content to apply himself to solution of the mathematical and astronomical problems that presented themselves to him, seeking only to achieve greater precision in the derivation of parameters and thus to obtain a better understanding of the relevant phenomena. His main contribution must thus be seen in the comprehensiveness of his work, as in his book on astrolabe construction, and in his continual attempts to formulate concepts like prayer times in mathematical terms. His attraction to sophisticated computational problems thus led him to consideration of more general theoretical questions.

BIBLIOGRAPHY

The most recent survey of Bīrūnī's works and biographical sources is that of E. S. Kennedy, "Al-Bīrūnī (or Berūnī), Abū Rayḥān (or Abūʾl Rayḥān) Muḥammad ibn Aḥmad," in *Dictionary of Scientific Biography* II, New York, 1970, pp. 148-58, and should be consulted by any serious reader. A considerably shorter essay by G. Saliba in *Dictionary of the Middle Ages* II, New York, 1983, pp. 248-51, covers some issues not treated by Kennedy and should also be consulted. The main bibliographical essay is still that of D. J. Boilot, "L'œuvre d'al-Beruni. Essai bibliographique," *Mélanges de l'Institut dominicain d'études orientales* 2, 1955, pp. 161-256; 3, 1956, pp. 391-96.

See also S. Ḥ. Naṣr, *Ketāb-šenāsī-e tawṣīfī-e Abū Rayḥān Bīrūnī*, Tehran, 1352 Š./1973; A. Saʿīd Khan, *Ketāb-šenāsī-e Abū Rayḥān Bīrūnī*, Pers. tr. ʿA. Ḥabībī, Tehran, 1352 Š./1973.

On the theoretical issues raised in this article see W. Harmer and M. Schramm, "Al-Bīrūnī and the Theory of the Solar Apogee. An Example of Originality in Arabic Science," in *Scientific Change*, ed. A. C. Crombie, London, 1963, pp. 206-18. A. Jouschkevitch, *Les mathématiques arabes (VIII^{e}-XV^{e} siècle)*, Paris, 1976. E. S. Kennedy et al., *Studies in the Islamic Exact Sciences*, Beirut, 1983. R. Rashed, *Entre arithmétique et algèbre*, Paris, 1984.