



ABU'L-WAFĀ BŪZJĀNĪ

ABU'L-WAFĀ' MOḤAMMAD B. MOḤAMMAD **BŪZJĀNĪ**, mathematician and astronomer, b. Wednesday, on the new moon of Ramaẓān, 328/10 June 940, at Būzjān in the region of Nīšāpūr. He studied arithmetic under his paternal uncle, Abū 'Amr Moğāzeli, and his maternal uncle, Abū 'Abdallāh MoḤammad b. 'Anbasa, presumably at Būzjān. He moved to Iraq at the age of nineteen (348/959-60) and soon became a leading mathematician and astronomer at the Buyid court in Baghdad. He is known to have made observations there in A.D. 974 (Bīrūnī, *al-Qānūn al-Mas'ūdī*, Hyderabad, 1954-56, II, pp. 640, 654-55, 677; and *Taḥdīd al-amāken*, ed. P. Bulgakov, Cairo, 1964, p. 301) and in 976 (*Qānūn* II, p. 658). Bīrūnī asserts (*Taḥdīd*, p. 100) he knows that Abu'l-Wafā' made most of his observations in Baghdad at the Bāb al-Tebn, during the reign of 'Ezz-al-dawla (356-67/967-78), in A.H. 365-66/Yazdejerdī 345-46; these two pairs of years overlap between 22 March 976 and 18 August 977. Abu'l-Wafā' was also associated with the observatory established by Šaraf-al-dawla (see *Abū Sahl Kūhī*). In one of the few recorded medieval attempts to establish the difference in longitude between two places, he observed the lunar eclipse of 24 May 997 at Baghdad, while Bīrūnī observed it in K̄vārazm. According to Ebn al-Qeftī (*Ta'riḳ al-ḥokamā'*, ed. J. Lippert, Leipzig, 1903, p. 288), he remained in Baghdad until his death on 3 Raġab 388/1 July 998. This date seems more likely than Ebn al-Aṭīr's report (IX, p. 137), repeated by Ebn Kallekān (tr. de Slane, III, pp. 320-21), that he died in 387/997-98. Among his pupils, the most illustrious was Bīrūnī's teacher, Abū Naṣr Maṣṣūr.

Abu'l-Wafā' was certainly one of the greatest mathematicians ever born in



Persia. Especially imaginative was his work in plane and solid geometry, in computing sine-tables to an interval of $0;30^\circ$, and in simplifying the solution of problems in spherical trigonometry. In astronomy the new parameters established by his observations continued in use for centuries after his death. It is greatly to be regretted that so little of his work on algebra has as yet been discovered. Of his great volume of writing, much has been lost or mutilated.

The following works are listed by Ebn al-Nadīm (*Fehrest*, p. 283) and repeated, in part, by Ebn al-Qeftī: 1. *Ketāb la-manāzel fī mā yaḥtāj elayh al-kottāb wa'l-'ommāl men 'elm al-ḥesāb* (“Book of the stations on what scribes and secretaries need of the science of calculation”). Its seven sections (“stations”) are devoted respectively to proportion, multiplication and division, operations of measurement, operations of taxation, operations of apportionments, money exchange, and mercantile transactions. It was written some time between ‘Azod-al-dawla’s assumption of the title Tāj-al-mella (368/979) and that ruler’s death (8 Šawwāl 372/26 March 983); see A. S. Ehrenkretz, “The *Taşrīf* and *Taşīr* Calculations in Mediaeval Mesopotamian Fiscal Operations,” *JESHO* 7, 1964, p. 46, n. 1. Ed. in A. Sa’idān, *‘Elm al-ḥesāb al-‘arabī*, ‘Ammān, 1971, pp. 64-368. Specialized studies on parts of the treatise include: P. Luckey, “Beiträge zur Erforschung der islamischen Mathematik. II,” *Orientalia* N.S. 22, 1953, pp. 166-89 (especially 175-79); M. I. Medovoy, “Ob odnom sluchae primeneniya otritsatel’nykh chisel u Abu-l-Vafy,” *Istoriko-matematicheskie issledavaniya* 11, 1958, pp. 593-98; idem, “Ob arifmeticheskom traktate Abu-l-Vafy,” *Ist.-mat. issled.* 13, 1960, pp. 253-324; Ehrenkretz, “The Kurr System in Medieval Iraq,” *JESHO* 5, 1962, pp. 309-14; idem, “Al-Būzajānī (A.D. 939-997) on the ” Ma’sīr,” *JESHO* 8, 1965, pp. 90-92; idem, “*Taşrīf*.” (See also below, no. 13.) 2. *Tafsīr ketāb al-Ḳvārazmī fī'l-ḵabr wa'l-moqābala* (“Commentary on Ḳvārazmī’s *Algebra*”), lost. 3. *Tafsīr ketāb Dīūfanṯes fī'l-ḵabr* (“Commentary on Diophantus’ *Algebra*”), lost. 4. *Tafsīr ketāb Ebarḵos fī'l-ḵabr* (“Commentary on Hipparchus’ *Algebra*”), lost. Although Hipparchus did not write an *Algebra*, Ebn al-Nadīm also refers to Abu'l-Wafā’s translation and commentary on the work elsewhere (p. 269); cf., however, Ebn al-Qeftī (p. 70) and Ḥājjī Kalīfa (*Kašf al-ẓonūn* [Leipzig] V, p. 73), where the original work is ascribed to Aristippus. 5. *Al-Modḵal elā al-aretmāṯiqī* (“Introduction to arithmetic”); a manuscript is extant at Rampur (Sezgin, *GAS* V, p. 403). 6. *Ketāb fī mā yanbaḡi yoḥfaz qabl ketāb aretmāṯiqī* (“On what one must memorize before studying [Diophantus’] *Arithmētikē*”), lost. 7. *Ketāb al-barāhīn ‘alā al-qazāyā allatī esta‘mala Dīūfanṯes fī ketābehe wa ‘alā mā esta‘malahu howa fī'l-tafsīr* (“Proofs of the theorems which Diophantus used in his book and what he [Abu'l-Wafā] used in the commentary”), lost. 8. *Ketāb*



esteḳrāj żel' al-moka “*ab be māl māl wa mā yatarakkab menhomā* (“Derivation of the side of a cube, of the square of a square, and of what is composed of these two”), lost. 9. *Ketāb ma'refat al-dawā'er men al-falak* (“Knowledge of the circles of the sphere”); this may be no. 20, below. 10. *Ketāb al-kāmel* (“The perfect book”); this is probably the *Ketāb al-maġestī* (*Almagest*), which is not noticed by Ebn al-Nadīm but is mentioned after *Ketāb al-kāmel* by Ebn al-Qeḫṫī; see also Abu'l-Faraġ, *Ta'riḳ moḳtaṣar al-dowal*, Beirut, 1958, p. 181. According to Ebn al-Nadīm the *Ketāb al-kāmel* (like the *Almagest*) consists of three books dealing, respectively, with preliminaries to the study of the motions of the celestial bodies (*Almagest*: trigonometry), the motions themselves (*Almagest*: applied spherical trigonometry), and the things which concern these motions (*Almagest*: planetary theory). An imperfect copy of the *Almagest* survives in Paris. A misinterpretation of one section of it led L. A. Sédillot to assert that Abu'l-Wafā' had discovered the variation of the moon's motion (“Découverte de la *variation*, par Aboul-Wafā, astronome du X^e siècle,” *JA* sér. 2, 16, 1835, pp. 420-38). This claim led to a long controversy, which was summed up and concluded against Sédillot's thesis by Carra de Vaux (“L'Almageste d'Abū 'lWéfa alBūzdġāni,” *JA* sér. 8, 19, 1892, pp. 408-71). On the spherical trigonometry in the *Almagest*, a field to which Abu'l-Wafā' made important contributions, see M. Delambre, *Histoire de l'astronomie du moyen âge*, Paris, 1819, pp. 156-66; and P. Luckey, “Zur Entstehung der Kugeldreiecksrechnung,” *Deutsche Mathematik* 5, 1940, pp. 405-46. On his instruments see L. A. Sédillot, *Mémoire sur les instruments astronomiques des Arabes*, Paris, 1842, pp. 195-97. Bīrūnī often cites the *Almagest*; he distinguishes it from the *Ziġ* in his *On Shadows* (the second treatise in his *Rasā'el*, Hyderabad, 1948, p. 43). 11. *Ziġ al-wāzeḩ* (“Lucid astronomical tables” = E. S. Kennedy, *A Survey of Islamic Astronomical Tables*, Philadelphia, 1956, no. 73). Now lost, it apparently was extremely influential. Its elements were used in a *Ziġ al-'alā'ī* (Kennedy, no. 42), whence they entered the *Ziġ al-šāmel* (Kennedy, no. 29). The latter may be identical with Aṫīr-al-dīn's *Ziġ al-molaḳkaṣ 'alā al-raṣad al-'alā'ī* (ca. 638/1240; Kennedy, nos. 40, 56; Sezgin, *GAS* V, pp. 324-25) and Cyriacus' *Ziġ al-ġadīd* (ca. 885/1480; Kennedy, no. 81).

In a further passage (p. 266), Ebn al-Nadīm attributes the following to Abu'l-Wafā': 12. A *Commentary* on Euclid's *Elements* (Ebn al-Qeḫṫī, p. 64; *Kaṣf al-ẓonūn* [Leipzig] I, p. 382), lost. Ebn al-Qeḫṫī adds: 13. *Ketāb al-'amal bi'l-ġadwal al-settīnī* (“Operating with a sexigesimal table”); this work is probably part of the *Ketāb al-manāzel*. Ebn Ḳallekān (tr. de Slane, III, p. 320) adds: 14. *Ketāb fī esteḳrāj al-awṫār* (“On the derivation of chords”). See F. Woepcke, “Sur une



mésure de la circonférence du cercle, due aux astronomes arabes, et fondée sur un calcul d'Abou'l Wafâ," *JA* ser. 5, 15, 1860, pp. 281-320. 15. *Ketāb fī mā yaḥtāġ elayh al-ṣāne' men a'māl al-handasa* ("On what artisans need of geometrical constructions") in thirteen chapters (*Kašf al-ẓonūn* [Leipzig] V, p. 172); the first seven chapters are preserved in a ms. in Milan and were translated in H. Suter, *Beiträge zur Geschichte der Mathematik bei den Griechen und Arabern*, Erlangen, 1922, pp. 94-109. The entire work survives in a ms. which once belonged to Uluġ Beg and is now in Istanbul. It is translated in S. A. Krasnova, "Abu-l-Vafa al-Buzdzhani, Kniga o tom, chto neobkhodimo remeslenniku iz geometricheskikh postroeniĭ," *Fiziko-matematicheskie nauki v stranakh* 1, 1966, pp. 42-140. A Persian translation was analyzed in F. Woepcke, "Analyse et extrait d'un recueil de constructions géométriques par Abou'l Wafâ," *JA* sér. 5, 5, 1855, pp. 218-56, 310-59. (See also Storey, II/1, pp. 2-3.) 16. *Resāla fī tarkīb 'adad al-wafq fī'l-morabba'āt* ("Epistle on constructing magic squares"). 17. *Fī mesāḥat al-moṭallatāt* ("On the area of triangles"), a response to a question put by Abū 'Alī Ḥasan b. al-Ḥāret. This is referred to by Abū Naṣr Maṣṣūr in his *Resāla fī ma'refat al-qosī al-falakīya* (eighth treatise in his *Rasā'el*, Hyderabad, 1948, p. 2). 18. *Resāla al-areṭmāṭiqī* ("Epistle on arithmetic"). 19. *Resāla fī'l-nesab wa'l-tarīfāt* ("Epistle on proportions and determinations"). 20. *Resāla fī eqāmat al-borhān 'alā al-dawā'er men al-falak* ("Epistle on establishing proof on the basis of the circles of a sphere"), addressed to Abū 'Alī Aḥmad b. 'Alī b. al-Sakr; published as the fifth treatise in *al-Rasā'el al-motafarreqa fī'l-hay'a* (Hyderabad, 1948) and expounded by N. Nadir, "Abū al-Wafā' on the Solar Altitude," *The Mathematics Teacher* 53, 1960, pp. 460-63. 21. *Barāhīn al-a'māl al-handasīya* ("Proofs of geometrical constructions"); extant in a Persian translation with a commentary by Moḥammad Bāqer Zayn-al-'ābedīn. 22. *Resāla fī jam' azlā' al-morabba'āt wa'l-moka'abāt* ("Epistle on adding together the sides of squares and cubes"). I owe to my colleague, Professor G. Toomer, the information that Abu'l-Wafā' in this epistle answers a question put by Abū Beṣr al-Ḥasan b. Sahl, and that he mentions as his patron al-Mo'ayyed al-Manṣūr, who is probably 'Azod-al-dawla's brother and regent in Isfahan, Mo'ayyed al-dawla (366-73/977-83). For some further works in manuscript, see *GAS* V, pp. 324-25.



BIBLIOGRAPHY

See also A. P. Youschkevitch in *Dictionary of Scientific Biography* I, New York, 1970, pp. 39-43. *ET*² I, p. 159.