



ĠIĀṬ-AL-DIN JAMŠID MAS‘UD KĀŠĀNI

ĠIĀṬ-AL-DIN JAMŠID MAS‘UD KĀŠĀNI (or Kāši), astronomer and mathematician (b. in Kāšān in ca. 770- 80/1368-78; d. in Samarqand 19 Ramaẓān 832/22 June 1429). We know nothing of Kāšāni’s early life save that his father must have been an accomplished astronomer and mathematician in order to understand the anecdotes that Kāšāni related to him in his letters, describing his life at the court (tr. in Bagheri, E. Kennedy, 1960; Tk. and Eng. tr. in Sayili, 1960b). The first dated event in Kāšāni’s life was his observation of a lunar eclipse at Kāšān on 12 Ḍu’l-ḥejja 808/2 June 1406, the first in a triplicity of observed lunar eclipses that he compared in his *Zij-e k āqāni* to a triplicity observed by Ptolemy. In the next year, on 21 Ramaẓān 809/1 March 1407, he completed, presumably still in Kāšān, his *Sollam al-samā’* on the sizes and distances from the earth of the planets and stars; he dealt with the distances of the planets again in the *Zij-e k āqāni*. Kāšāni dedicated the *Sollam al-samā’* to an as yet unidentified vizier named Kamāl-al-Din Maḥmud.

A few years later, he dedicated a Persian work on cosmology, the *Moktaṣar dar ‘elm-e hay’at*, to Sultan Jalāl-al-Din Eskandar Solṭān (q.v.), a grandson of Timur and the governor of Fārs, who was put to death at Šāhroḳ’s command in 818/1415 (Ḥāfeẓ-e Abru, p. 596); a fragmentary manuscript in the British Library was copied for Eskandar in 813-14/1410-11 (Rieu, *Persian Manuscripts*, p. 869).



Shortly after this, he received for the first time the patronage of another grandson of Timur, Oloġ Beg (r. 850-53/1447-49), the son of Šāhroġ and the ruler of Samarqand, for whom he completed an astronomical table (*zij*) in 816/1413-14 (*Zij-e k̄āqāni dar takmil-e Zij-e il-k̄āni*), which was intended to correct Našir-al-Din Ṭusi’s *Zij-e il-k̄āni* (Kennedy, 1956a, pp. 127-28, no. 20, abstracted on pp. 164-66; idem, 1998). The Istanbul manuscript (Aya Sofya 2692) was copied in the year of its composition, perhaps by the author himself. Kāšāni divided his *Zij* into six treatises (*maqāla*); each treatise is made of an introduction followed by a couple of chapters (*bāb*), except for the first one that is comprised of the introduction and four chapters. This important *zij* has not yet been published, although several sections of it have been studied by Edward Stewart Kennedy and some of his students. The first treatise is devoted to the description of calendars (*dar ma’refat-e tawāriġ-e mašhur*): the Seleucid, Hejri, and Yazdegerdi; the Chinese-Uighur; and the Jalāli; and their festivals. The description of the Chinese-Uighur calendar was discussed by Edward Kennedy (1964; repr. in D. A. King and M. H. Kennedy, eds., pp. 652-60; see also van Dalen, Kennedy, and Saiyid). The second treatise deals with the determination of the trigonometric, declination, and ascension functions and includes a geographical table of the longitudes and latitudes of more than five hundred cities (*dar ma’refat-e jayb o sahm o zell o mayl o maṭāle’ o dekr-e ṭul o ‘arż-e boldān*); the geographical table was published by Edward and M. H. Kennedy (1987). The double-argument tables for the equations of planetary longitudes (3rd treatise: *dar ma’refat-e mawāze’-e kawākeb dar ṭul o ‘arż wa tawābe’-e ān*) were analyzed by Mark Tichenor (1967, repr. in D. A. King and M. H. Kennedy, eds., pp. 122-24), and his method for computing planetary ephemerides using second order differences by Edward Kennedy (1962, repr. in D. A. King and M. H. Kennedy, pp. 522-25). Kāšāni’s parallax theory was studied by Edward Kennedy (1956b, repr. in D. A. King and M. H. Kennedy, pp. 164-84), who also (1985) elaborately described the fourth treatise of the *Zij* on the various operations of spherical astronomy (*dar ma’refat-e qessi-e mašhura*), as well as the fifth treatise on the determination of the ascendant (*dar ma’refat-e tāle’ az ma’lumāt-e moġtalefa*; Kennedy, 1995-96; for a section of this latter see also Kennedy and Debarnot). In the last treatise (*dar bāqi-e a’māl-e nojumi*) the computation of astrological entities is discussed. Kāšāni’s method of determining the cusps of the astrological places was explained by Edward Kennedy (1994). Finally, a part of the *Zij* dealing with planetary stationary points and retrogressions was translated into Sanskrit at Jayapura in about 1730 (see Pingree, 2000).



Kāšāni's next work was a short Persian treatise on astronomical instruments entitled *Resāla dar šarḥ-eālāt-e rašad*, which he completed in 818/ January 1416 and dedicated to Solṭān Eskandar, perhaps the son of the Qara Qoyunlu prince Qara Yusof, who ruled western Persia from 823/1420 till 841/1438; it describes the triquetum, the armillary sphere, the equinoctial ring, the double ring, the Faḳri sextant, the azimuth-altitude instrument and an instrument employing sine and versed sine, all well-known instruments. At the end Kāšāni calls himself a physician (*ṭabīb*). A facsimile of the unique manuscript with an English translation and a commentary was published by Edward Kennedy (1961, repr. in D. A. King and M. H. Kennedy, eds., pp. 394-404). A month later, on 10 818/ 10 February 1416, Kāšāni finished at Kāšān the first version of his *Nozhat al-ḥada'eq* on the planetary equatorium that he invented; he corrected and supplemented this first version at Samarqand in 829/June 1426. Edward Kennedy dealt with the various applications of the plates of this instrument in several articles (1947, 1950, 1951a, 1951b, 1952; all reprinted in D. A. King and M. H. Kennedy, eds., pp. 448-80). An anonymous Persian summary of the

Nozhat al-ḥada'eq, which is dedicated to the Ottoman Sultan Bāyazid II (1481-1512), was published in facsimile with an English translation by Edward Kennedy in his edition/translation of *Ṭabaq al-manāṭeq...* (1960). The astronomical problems that can be solved with these computing instruments are the determination of the times of the conjunctions of the planets, of the true longitudes of the sun, the moon, and the planets, of the latitudes of the moon and the planets, and of the times and magnitudes of lunar eclipses.

In about 1412, when Olōg Beg began to construct the observatory at Samarqand, Kāšāni was called to his court and was soon considered the leading scientist of the group assembled. Kāšāni explains the process of how he reached this favorable position at the court in two letters that he wrote to his father in Kāšān, the first after he had been at Samarqand for two years (i.e., in ca. 1423; tr. with comm. by Bagheri) and the second slightly later (tr. with comm. by Kennedy, 1960, repr. in D. A. King and M. H. Kennedy, eds., pp. 722-44; ed. and tr. into Turk. and English by Sayili, 1960b). These letters provide a fascinating picture of the interactions between the prince, his scientists, and his workmen as they designed and constructed the observatory.

Besides his work on the observatory Kāšāni was also engaged in composing his truly extraordinary works on mathematics. In 827/July 1424 he completed his *al-Resāla al-moḥiṭiya*, in which he computed the value of p to



nine sexagesimal or sixteen decimal places. This was edited (incom- pletely) and translated into German by Paul Luckey (1953); a facsimile of a complete manuscript and a translations into Russian were published by B. A. Rosenfeld, V. S. Segal, and A. P. Yushkevich. On 3 Jomādā I 830/2 March 1427 Kāšāni completed his masterpiece, the *Meftāḥ al-ḥesāb*, which he dedicated to Olōg Beg. This work consists of five books: the first is on the arithmetic of integers, and includes innovative sections on root-extraction and binomial expansions (see Luckey, 1948); the second is on the arithmetic of fractions; the third is on sexagesimal fractions, including a chapter on the root-extraction of sexagesimals (see Dakhel, 1960), and on decimal fractions, which Kāšāni claims to have invented, apparently being unaware of their use by Aḥmad b. Ebrāhim Oqlidesi in 952-53 (Saidan, pp. 481-85); the fourth is on the mensuration of geometrical figures, including sine-tables, specific gravity, and a chapter on the geometry of arch façades (*morqarnas*; see Dold-Samplonius, 1992, 1996); and the fifth is on algebra, including square root extractions of polynomials. The *Meftāḥ al-ḥesāb* was lithographed in Tehran in 1889; it was edited and commented on in Arabic by Aḥmad Sa‘id Damardāš and Moḥammad H‘amdi H‘efni; a facsimile of the Leiden manuscript with a Russian translation was published by Rosenfeld, Segal, and Yushkevich; and it was edited with notes by Nader Nābolosi. The contents of the first three books, on arithmetic are described by Paul Luckey in his edition/translation of *al-Resāla al- moḥiṭiya* (1951).

Finally, Kāšāni computed the value of the sine of 1° by an iterative solution of a cubic equation. He described this computation in a *Resālat al-watar wa al-jayb*, which, apparently, he did not live to finish; it was completed, it is said, by Qāzizāda Rumi, his colleague at Olōg Beg’s court. The *Resāla* exists in manuscript form and was lithographed at Tehran in 1888, but is best known from Miram Ūelebi’s commentary of 1499 on Olōg Beg’s *Zij-e jadid* (see Sédillot; see also Aaboe for an exposition of the method). This computation appears in Sanskrit texts from the 17th and 18th centuries (see, e.g., Pingree, 1978).

There exist in manuscripts a number of other treatises in Arabic and in Persian attributed to Kāšāni; none of them has as yet been studied or published, but their names can be found in Kennedy (tr, *Ṭabaq al-manāteq*, pp. 6-7 and Youschkevitch and Rosenfeld, p. 261). Without taking them into consideration one must conclude on the basis of the works already discussed that Kāšāni was one of the most brilliant mathematicians of medieval Islam. A few lines of poetry are ascribed to him by Moḥammad.



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